

Answer the following questions. Calculators and mobile telephones are not allowed.

1. The region R is bounded by the following curves: $y = \ln x$, $y = 0$ and $x = e$. Sketch the region R and find the volume of the solid generated by revolving the region R about the line $y = -1$. (5 points)

2. Let $f(x) = \ln \frac{e^x}{1 + e^x}$. Determine whether or not f^{-1} exists. If it exists find $f^{-1}(x)$ and state the domain and range of $f^{-1}(x)$. (4 points)

3. Find $g'(x)$ if $g(x) = \frac{1}{2} \left(\frac{1}{2} \ln \frac{x+1}{x-1} + \tan^{-1} x + \pi^x \right)$. (4 points)

4. (a) Solve the equation: $\cos^{-1} x = \sec^{-1} x$. (4 points)

(b) Prove the identity: $\cos^{-1}(-x) = \pi - \cos^{-1} x$, $|x| < 1$. (4 points)

5. Evaluate the following integrals: (4 points each)

(a) $\int \frac{1}{1 + \sec x} dx$

(b) $\int \frac{x^5}{x^4 - 16} dx$

(c) $\int \sqrt{1 + e^{2x}} dx$

(d) $\int_0^{\frac{\pi}{2}} \frac{\cot x}{1 + \ln(\sin x)} dx$

6. Evaluate $\lim_{x \rightarrow 0} \frac{\ln(\cos 2x)}{\ln(\cos 3x)}$, if it exists. (4 points)

7. Find the equation of the parabola with vertex $V(2, 5)$ passing through the point $P(3, 6)$ and whose directrix is parallel to the x -axis. (4 points)

8. Answer ONLY ONE PART (either (a) or (b))

(a) Find the arc length of the curve defined parametrically by the equations $x(t) = -\cos t + 5$, $y(t) = \sin t + t - \frac{\pi}{2}$ for $0 \leq t \leq \frac{\pi}{2}$. (5 points)

(b) Given the polar equation $r = \frac{2}{\pi}\theta + 1$, $0 \leq \theta < 2\pi$; (5 points)

(i) Sketch the graph of the curve and label at least three points on the graph.

(ii) Find the area of the region in the second quadrant bounded by the curve.